



A levels Mathematics (9709) made easy with ClassWiz (Fx-991EX)

Vectors

The Basics

Q1. With respect to the Origin O, the points A, B, C, D have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix} 5 \\ -2 \\ -2 \end{pmatrix}, \overrightarrow{OC} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \overrightarrow{OD} = \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}$$

Calculate

- 1) AB
- 2) CD

Solution:

Understanding ClassWiz's operations

You can easily do vector-based calculations with the all new Casio ClassWiz FX-991EX.

Follow the steps below

- a) Put your ClassWiz on the **Vector** mode.

"Right after you select Vector mode, the display asks you to define **Vectors**.
(Look at the figures on right of the page for help)
Here we are going to save the vectors **OA**, **OB**, **OC**, and **OD** as **VctA**, **VctB**,
VctC, and **VctD**"

- b) Select 1. Select Dimension (whether it has 2, or 3 coordinates) and add the coordinates. In this case, they are as follows (Results showed in fig1, and fig2).



VctA

Dimension?

Fig1



Fig2

Select 2~3

1

- c) Press **OPTN**, you'll see this menu (on right), and select **1** (Define Vector) following with **2** to define the vector **OB** (select dimensions, and add coordinates).
- d) Repeat the step c.



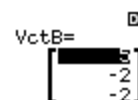
Define Vector

1:VctA 2:VctB
3:VctC 4:VctD

1:Define Vector
2:Edit Vector
3:VctA 4:VctB
5:VctC 6:VctD

VctB
Dimension?



Select 2~3










i) In theory we know that for a vector \overrightarrow{AB} , the arithmetic expression would be $\overrightarrow{OB} - \overrightarrow{OA}$.

Now after you have saved all the vectors, it's time for the Calculation.

Press  , and select 'Vector Calc' by pressing 3. (See fig 3)

Again press  , and for the arithmetic operations above write

(    )

the expression as **VctB – VctA**.

$$\text{VctB} - \text{VctA} = \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix}$$


1

Woohoo! You just learned the very basics for vector calculation.

Now try finding the vector \overrightarrow{CD} yourself.

(Ans. ClassWiz's operations for vector \overrightarrow{CD} are:      )

Now let's see How the functions of Casio's all new ClassWiz (Fx-991EX) series help solve A Levels Mathematics (P3) Vectors' questions.

 As an example, and to showcase the mind-blowing functions, we have chosen the following question from CIE past papers.

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10 The lines l and m have vector equations

$$\mathbf{r} = \mathbf{i} - 2\mathbf{k} + s(2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) \quad \text{and} \quad \mathbf{r} = 6\mathbf{i} - 5\mathbf{j} + 4\mathbf{k} + t(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$$

respectively.

(i) Show that l and m intersect, and find the position vector of their point of intersection. [5]

(ii) Find the equation of the plane containing l and m , giving your answer in the form $ax + by + cz = d$. [6]

As you can see, this question carries 11 marks (in total). What if you can solve the question quickly with 100% accuracy? Seems unlikely, No? Well, No.

With all new **Casio ClassWiz (Fx-991EX)** now you can actually solve the question speedily and maintaining 100% accuracy (okay... at least 99.99% just like safeguard).

Let's start!



Fig 3

1:Define Vector
2>Edit Vector
3:Vector Calc

3

□

Vector



(i)

Show that l and m intersect, and find the position vector of their point of intersection. [5]

Now for such a question, you form 3 equations for x , y , and z in terms of s , and t (in this case), and solve them simultaneously to get the points of intersection (if present).

a) For such a case the 3 simultaneous equations would be:

$$X = 1 + 2s$$

$$Y = s$$

$$Z = -2 + 3s$$

$$X = 6 + t$$

$$Y = -5 - 2t$$

$$Z = 4 + t$$

Here we have 2 unknowns to find i.e., s , and t so we'll choose any 2 equations and solve them.

Make three equations ($X = X$, $Y = Y$, $Z = Z$). The resulting equations would be as follows

$$2s - t = 5 \quad \text{-(i)}$$

$$s + 2t = -5 \quad \text{-(ii)}$$

$$3s - t = 6 \quad \text{-(iii)}$$

Now with your **ClassWiz (Fx-991EX)** go the **Equation/Func** mode; Select **Simul Equation**; Enter the number of unknowns (in this case 2), and press = to get the result.

(Follow this: **MENU** **(←)** **1** **2** **2** **=** **1** **=** **5** **=** **1** **=** **2** **=** **5** **=** **=** **=**)



1:Simul Equation
2:Polynomial

Simul Equation
Number of
Unknowns?
Select 2~4

$$\begin{cases} 2x - 1y = 5 \\ 1x + 2y = -5 \end{cases}$$

$$x = 1$$

$$y = -3$$

At last, we have calculated the value of s , and t (disguised as x , and y). Put these value in 3rd simultaneous equations - you have used only first two equations – to confirm the answer.

Put the value of s , or t in line l , or m to **get the position vector of their point of Intersection**.



(ii)

Find the equation of the plane containing l , and m , giving your answers in the form $ax + by + cz = d$

As we know, the plane of an equation is in the form $\vec{r} \cdot \vec{n} = \vec{n} \cdot \vec{OA}$ – where \vec{OA} is any point on the plane.

To form this equation, we need to find the normal of the plane, and for the normal we know the **Cross Product** of direction vectors of 2 lines (that lies on the plane) gives us the **normal** to the plane.

Taking **V1** as direction vector of **line l** and **V2** as direction vector of **line m**, we get:

$$\mathbf{V1} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$$

$$\mathbf{V2} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$$

Now open **ClassWiz (Fx-991Ex)** and define **V1** as **VctA**, and **V2** as **VctB**.

(Follow these

buttons: **MENU** **5** **1** **3** **2** **=** **1** **=** **3** **=** **OPTN** **1** **2** **3** **1** **=** **=** **2** **=** **1** **=**)

Now to find the **Cross-Product** (or **normal**), type **VctA x VctB** (refer/see to the guide/picture as follows)

$$\text{VctA} \times \text{VctB} = \begin{bmatrix} 7 \\ 1 \\ -5 \end{bmatrix}$$

7

Finally, you have calculated the value for the **normal**. $\vec{r} \cdot \vec{n} = \vec{n} \cdot \vec{OA}$.

Now you need to calculate the value for the **dot-product** of **Normal**, and \vec{OA} .

For a general point on the plane, put s in **line l** or t in **line m** equal to 0.

Here I am putting $s = 0$, and the point I get is $\mathbf{i} - 2\mathbf{k}$. I'll define this point as **VctC**.

Now, Select **OPTN**, **OR**, and **1** for **VctAns** (The vector of normal you just calculated) [**OPTN** **1**]

Again **OPTN**, **OR**, and select **2**, for **Dot Product**. [**OPTN** **2**]

Once again **OPTN**, and select **5** for **VctC**. [**OPTN** **5**]

$$\text{VctAns} \cdot \text{VctC}$$

And press **=**

You get:

$$\text{VctAns} \cdot \text{VctC} = 17$$



Now you can rewrite the equation as

$$\vec{r} \cdot (7\vec{i} + \vec{j} - 5\vec{k}) = 17$$

And simplifying it in the form $ax + by + cz = d$

i.e., $7x + y - 5z = 17$

Further carrying on the question, **what if we were required to find the *Acute angle* between line l , and line m ?**

To find the angle between 2 intersecting lines, you can use **ClassWiz's Angle** function.

You have already saved the direction vectors of line l , and line m as **V1**, and **V2** in **VctA**, and **VctB**. We'll make use of that.

Generally, the formula we use is $\cos \theta = \frac{V1.V2}{|V1|.|V2|}$

So how can you find the angle with **ClassWiz (Fx-991EX)**?

Refer to the following guide:












While still in the *vector* mode, Select  ,  , and  .   

Then type in **VctA** , **VctB** and press  .      

At last! You have calculated the angle between the **line l** , and **line m** .

Angle(VctA, VctB)
70.89339465

Miscellaneous commands

1. Press  followed by  for **Reset**.
2. Press  followed by  for direct **answer** in **decimals** rather than **fractions**.
3. Press  followed by  for **conversions**.
4. Press  followed by  for **constants**.
5. Press  followed by  , followed by  to choose between **Degree**, **Radian**, or **Gradian**.